Quantitative Literacy:
Thinking Between the Lines

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Chapter 5: Introduction to Probability

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Lesson Plan

- Calculating probabilities: How likely is it?
- Conditional probability
- Counting and theoretical probabilities: How many?
- More ways of counting: Permuting and combining
- Expected value and the law of large numbers: Don't bet on it

Learning Objectives:

- Distinguish between the different types of probability
- Calculate mathematical probabilities with:
  - Theoretical probability
  - Distinguishing outcomes
  - Probability of non-occurrence
  - Probability of disjunction
  - Probability with area

Chapter 5: Introduction to Probability
5.1 Calculating probabilities: How likely is it?

- If each outcome of an experiment is *equally likely*, the **probability** of an event is the fraction of favorable outcomes.

$$\text{Probability of an event} = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible equally likely outcomes}}$$

- A probability of an event is the fraction of favorable outcomes.

$$\text{Probability} = \frac{\text{Favorable outcomes}}{\text{Total outcomes}}$$

- A Probability must be between 0 and 1.
- The probability of an event is 0 $\Leftrightarrow$ the event can never occur.
- The probability of an event is 1 $\Leftrightarrow$ the event will always occur.

- **Example:** Suppose I flip two identical coins. What is the probability that I get two heads?
- **Solution:** There are four equally likely outcomes.

Nickel	Dime
H	H
H	T
T	H
T	T

$$P(HH) = \frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{1}{4}$$

Example: Suppose I have a 50-50 chance of getting through a certain traffic light without having to stop. I go through this light on my way to work and again on my way home.

To work	To home
Stop	Stop
Stop	Don't stop
Don't stop	Stop
Don't stop	Don't stop

1. What is the probability of having to stop at this light at least once on a workday?
2. What is the probability of not having to stop at all?

- **Solution:**

1. 50-50 chance: the probability of stopping at the light is ½ and the probability of not stopping is ½

$$\frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{3}{4}$$

2. One of the possible outcomes (Don't stop-Don't stop) corresponds to not having to stop at all:

$$\frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{1}{4}$$

$$\text{Probability} \text{of event } \textbf{not} \text{ occurring}$$
$$= 1 - \text{Probablity of event occurring}$$

- **Example:** There are several sections of English offered. There are some English teachers I like and some I don't. I enroll in a section of English without knowing the teacher. A friend of mine has calculated that the probability that I get a teacher I like is:

$$\mathrm{P(Teacher\ I\ like)} = \frac{7}{17}$$

What is the probability that I will get a teacher that I don't like?

- **Solution:** $$\mathrm{P(Teacher\ I\ don't\ like)} = 1 - \mathrm{P(Teacher\ I\ like)}$$
$$= 1 - \frac{7}{17} = \frac{10}{17}$$

- **Example:** Suppose we toss a pair of standard six-sided dice.

1. What is the probability that a we get a 7?
2. What is the probability that we get any sum but 7?

- **Solution:**

Red die	Green die
1	6
2	5
3	4
4	3
5	2
6	1

1. Probability of a 7

$$= \frac{6}{36} = \frac{1}{6} = 0.17 = 17\%$$

2. Probability of event **not** getting a 7

$$= 1 - \frac{1}{6} = \frac{5}{6} = 0.83 = 83\%$$

- The **disjunction** is the event that either A or B occurs. The probability of this disjunction:

$$\boxed{\mathrm{P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)}}$$

- **Example:** Suppose a librarian has a cart with 10 paperback algebra books, 15 paperback biology books, 21 hardbound algebra books, and 39 hardbound biology books. What is the probability that a book selected at random from this cart is an algebra book or a paperback book?

- **Solution:**

Let A be an algebra book and B be a paperback book.

Three probabilities: $P(A)$, $P(B)$, and $P(A \text{ and } B)$.

Altogether, there are $10+15+21+39=85$ books.

$$P(A) = \frac{31}{85}, \quad P(A) = \frac{25}{85}, \quad P(A \text{ and } B) = \frac{10}{85}.$$

$$\boxed{\begin{aligned} P(A \text{ or } B) &= P(A) + P(B) - P(A \text{ and } B) \\ &= \frac{31}{85} + \frac{25}{85} - \frac{10}{85} = \frac{46}{85} = 0.54 = 54\% \end{aligned}}$$

- **Example:** The surface area of Earth is approximately 197 million square miles. North America covers approximately 9.37 million square miles, and South America covers approximately 6.88 million square miles. A meteor falls from the sky and strikes Earth.

  What is the probability that it strikes North or South America?

- **Solution:** The total area covered by North and South America is $9.37+6.88=16.25$ million square miles.

$$\text{Fraction of the surface area of Earth} = \frac{16.25}{197} = 0.082 = 8.2\%.$$

5.1 Calculating probabilities: How likely is it?

- The **empirical probability** of an event is a probability obtained by experimental evidence.

$$\text{Probability} = \frac{\text{Favorable outcomes}}{\text{Total number of outcomes in the experiment}}$$

- **Example:** Suppose the city posted workers at the intersection, and over a five-week period it counted 16,652 vehicles passing through the intersection, of which 1432 ran a red light. Use these numbers to calculate an empirical probability that cars passing through the intersection will run a red light.
- **Solution:** 1432 out of 16,652 ran the red light.

$$\text{An empirical probability} = \frac{1432}{16{,}652} = 0.09 = 9\%$$

Learning Objectives:

- Understand
  - Conditional probability

5.2 Medical testing and conditional probability: Ill or not?

- A **conditional probability** is the probability that one event occurs given that another has occurred.
- **Example:** The accompanying table of data is adapted from a study of a test for TB among patients diagnosed with extra pulmonary TB .

	Has TB	Does not have TB
Test positive	446	15
Test negative	216	323

Calculate the conditional probability that a person tests positive given that the person has TB.

- **Solution:** $446 + 216 = 662$ people who have TB.

$$\text{P(Positive test given TB is present)} = \frac{\text{True positives}}{\text{All who have TB}} = \frac{446}{662}$$

$$= 0.674 = 67.4\%$$